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Review Article

A state-of-the-art review on the modeling and probabilistic approaches to analysis of power systems integrated with distributed energy resources

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ABSTRACT

Modern power systems are shifting toward decarbonization and incorporation of distributed energy resources (DERs) to replace fossil fuel generators. Although promising, DERs introduce uncertainty because of their intermittent nature. This study provides a comprehensive survey of current approaches for modeling system uncertainties and methods of analysis, particularly in the context of static voltage stability studies within modern power systems. Emphasis is placed on evaluating various models applied to different system random variables (RVs), focusing on their suitability for those particular RVs. Additionally, the study examines the characteristics and frameworks of prominent probabilistic methods (PM), evaluates their efficacy, and discusses static voltage stability analysis approaches, emphasizing solution structures and appropriate applications. It concludes by thoroughly reviewing both numerical and analytical PM methods and offering insights into their strengths and limitations. The provided comprehensive survey reveals that, considering system uncertainties, voltage stability studies have gained the most share, followed by small-signal stability studies, whereas the frequency stability studies have gained the least share.

1. Introduction

The core requirements of a power system are (a) the capacity to supply active and reactive power with adequate spinning reserve, (b) supplying power at minimum cost and ecological impact, and (c) maintaining quality standards, that is, constancy of frequency, constancy of voltage, and level of reliability [1]. This description details the conventional power system paradigm and its operational philosophy. Historically, power systems have relied on synchronous generators (SG) for crucial functions, including ensuring spinning reserve margins, maintaining frequency stability, and regulating bus voltages within stiff margins.

However, key developments in the power industry have modified these requirements and fundamentally revised operational, control, and analytical approaches. Deregulation has prompted the active participation of independent power producers (IPPs) and demand-side management [2], which necessitates optimal grid operation. There are new strict requirements, such as reduction in system losses, optimal generation, and cooperation of distributed energy resources (DERs) to minimize losses, reduce operational costs and maximize profits [3–6]. These new requirements aim to optimize the power system operation, reduce spinning reserves, and replace fossil fuel based generators with DERs. It also establishes energy markets by competitive bidding and fostering generational diversification.

Because DERs are intermittent and uncertain, their increasing prevalence in power systems increases the level of system uncertainties. Traditional deterministic methods of analysis that rely on predictable inputs, are inadequate for capturing the rapid and non-deterministic variations introduced by modern power system components, such as DERs and loads. This is largely because they overlook the uncertainty in their models and analysis methods, which leads to unreliable solutions [7]. Furthermore, the integration of DERs into power systems has been noted to significantly affect the voltage profile and stability of the network [8]. Over time, several approaches for studying different aspects of power systems with uncertainties have been proposed, such as possibilistic methods [9–11], interval estimation [12–15], robust satisficing [16–18], information–gap theory [19–21], and machine learning [22–25]-based methods, as shown in Fig. 1.

Probabilistic methods (PM) have emerged as prominent credible alternative solutions, offering insights into system responses to varying

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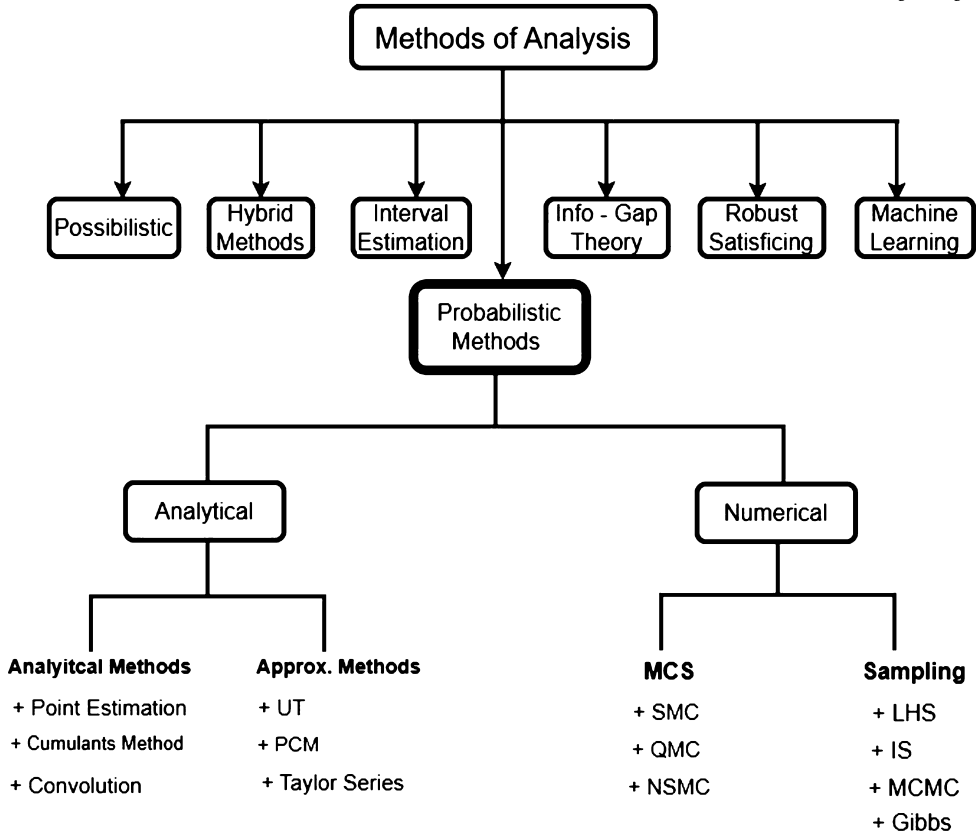

Fig. 1. Classification of methods of analysis of power systems.

inputs. The PM simplifies modeling and analysis while offering comprehensive information on the system uncertainties. This review focuses solely on PM owing to their simple solution algorithms, detailed probabilistic outputs, rapid execution speed, and ease of implementation, making them suitable for real-time online applications for control, operation, and engineering planning or design applications [26].

To enhance the effectiveness of PM, it is crucial to utilize suitable models for system uncertainties. Many reviews, such as [27–30] simply provided random variables (RVs) with their corresponding assigned models with the respective studies that used them. However, a few other surveys, such as [31,32] tend to use models and methods of analysis interchangeably. Given the breadth of research on power system analysis, researchers may encounter challenges in selecting appropriate models for power system studies involving particular RVs. In addition, PM has been developed from two divergent perspectives: numerical- and analytical-based approaches, as summarized in Fig. 1. Several reviews on this subject [28–30] tend to survey the entire spectrum of possible solution strategies, overlooking the need for a detailed analysis of the most promising PMs. Moreover, most reviews have provided only a few promising methods in the PM category. Therefore, this paper aims to address these challenges with the following research objectives:

1. To identify the sources of uncertainties in modern power systems and how they are currently being modeled.
2. To evaluate the suitability and efficiency of the applied RV models power system stability studies.
3. To assess effectiveness of existing probabilistic approaches to modern power system analysis involving uncertainties

To achieve these research objectives, an extensive literature review was performed using rigorous and strict methodology, as described in Section 3. Consequently, this review provides a state-of-the-art overview of statistical models and probabilistic methods for the stability analyses of modern power systems that involve uncertainties. The literature published between 2012 and 2023 was reviewed. The remainder of this paper is organized as follows. Section 2 provides a concise literature review, and Section 3 outlines the methodology used for the review, placing it in the context of existing literature. Section 4 presents the results and discusses the key findings. Finally, Section 5 provides the concluding remarks.

2. Literature review

2.1. Overview of some recent review studies

Table 1 summarizes the notable reviews conducted between 2012 and 2023, a period coinciding with the increased penetration and participation of DERs in power systems following the ratification of the Doha amendment to the Kyoto Protocol in 2012.

The list provided in Table 1 is not exhaustive but aims to illustrate the influence of existing studies on shaping the body of knowledge regarding the subject. This is inspired by observations in [103], in which the authors highlight the challenges of the integration of DERs in power systems and the perceived benefits accrued, while [33] offers a comprehensive examination of the system components that require modeling to reflect the influence of uncertainties on system performance.

Many reviews on statistical models for system uncertainties have been observed to simply list relevant studies, such as those in [28,29,33], lacking evaluation of their suitability, efficiency, and completeness. Additionally, while some reviews cover a broad range of analytical methods, they often lack a focused detailed treatment of the more promising PM. Furthermore, in some reviews, for instance, [31] and [32], the methods of analysis and models are used interchangeably. Moreover, most reviews have provided only a few promising methods in the PM category. Therefore, the aim of this study is to fill some of these gaps in the subsequent sections.

Table 1
A summary of some notable conducted literature reviews.

Ref	Year	Key findings / contributions		Methods Reviewed
[27]	2019	Provides:	Comparison of PMs Popular RV models	NBPM: MCS, QMC, MCMC ABPM: PEM, CM, PCM
[33]	2017	Presents:	RVs to be modeled Discussion of PMs	NBPM: MCS ABPM: PEM, CM, PCM
[28]	2016	Examines:	Uncertainty handling methods RV statistical models	NBPM: MCS, SMC, QMC ABPM: PEM, CM, UT
[30]	2018	Provides:	Common RV models Some research directions	MCS, & non-PMs ABPM: PEM, & hybrids
[31]	2019	Presents:	RVs requiring modeling A discussion of PPF methods	MCS, & hybrids ABPM: PEM, UT, CM
[29]	2017	Provides:	A list of uncertainty models Uncertainty handling methods	MCS, LHS, QMC ABPM: PEM, UT, CM
[34]	2012	Identifies:	Popular load models DGs assumed negative loads Common modeling software	Constant P, I, Z, & ZIP Exponential & polynomials Uncertainties ignored
[35]	2017	Provides:	Common load models Load parameter identification Static & dynamic models	ZIP, exponential, EPRI LOADSYN IM, Freq. dependent Ignores uncertainties
[36]	2022	Presents:	Uncertainty handling methods Areas of application RV models	NBPM: MCS, LHS, QMC ABPM: CM, PEM, UT Non-PM: Possibilistic & hybrids
[37]	2022	Examines:	Impact of solar PV on grid Mitigating solutions & modeling software	MATLAB, PSAT, GAMS & RTS
[38,39]	2023	Describes:	Voltage instability phenomena & indices Voltage instability as an optimization constraint	Several voltage stability indices Particle swarm optimization
[32]	2023	Discusses:	Methods of dealing with system uncertainties presented as probabilistic: frequency, voltage, transient and small signal stability analyses	MCS, PEM, CM, PCM, importance sampling

PCM (Probabilistic Collocation method); QMC (Quasi Monte Carlo); SMC (Sequential Monte Carlo); MCS (Monte Carlo Simulation); NBPM (Numerical Based PM); ABPM (Analytical Based PM); MCMC (Markov Chain Monte Carlo); CM (Cumulants Method); PEM (Point Estimates methods).

2.2. Uncertainties in modern power systems

Modern power systems are experiencing a heightened increase in the uncertainty levels arising from various sources. Table 1 presents some notable recent reviews, and Table 2 explores recent studies involving system uncertainties. Fig. 2, obtained from the surveyed literature and recent reviews provided in Tables 1 and 2, shows a wide spectrum of system components characterized by uncertainties with their corresponding models, as applied in various power system studies. A numerical variable is incremented for each RV if it is featured in a sampled paper, resulting in a numerical distribution, as shown in Fig. 2. It can be observed from the figure that DERs and loads continue to attract significant attention in the literature ahead of other system components.

Several approaches for studying the uncertainties in power systems have been proposed in the literature. These include probabilistic, possibilistic, interval estimation, robust satisficing, and information-gap-based methods. This review focuses on probabilistic methods (PM), owing to the simplicity of their solution algorithms and the ability to provide sufficient details in the form of probability density functions (PDFs) of system states, fast execution speed, and ease of implementation as compared to other methods [104–106].

PM needs to be fast, applicable for real-time control environments, or used as a fast screening tool for engineering planning or design applications [26]. For PMs to work well, models must be used to account for the system uncertainties. Therefore, this review provides a state-of-the-art on modeling and probabilistic methods of analysis of power system stability studies.

3. Methodology of the review

3.1. The inclusion/exclusion criteria

To select studies for review, the following criteria were employed for sieving through numerous published studies.

3.1.1. General criteria

All studies concerned with the modeling and analysis of power systems involving uncertain RV inputs were candidates for inclusion in the review, especially those performed and published between 2012 and 2023.

3.1.2. Specific criteria

- Studies involving the application of probabilistic methods to analyze power systems with any form of uncertainty.
- Studies involving the integration of distributed energy resources in power system analyses such as voltage, small-signal, transient, and frequency stability analyses.
- All studies that involved either wind turbine power generation, or solar PV power generation or both in any probabilistic stability study.

The review excluded editorials, all unpublished works, book reviews, technical reports, and works that did not use probabilistic methods, even when they were concerned with power system uncertainties.

Table 2
A survey on PM applications and approaches with corresponding uncertainty models.

Method	Form of Stability	Models Used
PCE	Small signal	solar: beta; wind: Weibull [40]; Load: exponential [41], uniform [42,43]
	Transient	Solar: Gaussian KDE [44], historical data [45]; wind: historical data [45]
	Voltage	SGs & loads: normal, DC-link: uniform [46]; Wind: Weibull [47–49]
	Frequency	Load: normal, generator inertia: uniform [50]
MCS	Small signal	Wind: Weibull [51–54]; load: normal [54,55], historical data [56] Solar: historical data [56]; generator: binomial [55]
	Transient	Load: normal [57,58]; CB: normal [59,60], wind: Weibull [57,61]; Solar: beta [57] Fault occurrence: historical data [57] fault location: Poisson [58–60]
	Voltage	Load: exponential [62], normal [62–67], GMM [68] SG & DERs dispatch: normal [69]; Fault issues: historical data [70] Wind: Weibull [63–68,71,72]; Solar: beta [65–68], Lognormal [63,64]
	Frequency	CB tripping: normal [73], load shedding: historical data [74]
CM	Small signal	Solar: normal [75], Wind: normal [75], load: normal [75], historical data [76] Wind: Weibull [77], GMM [71], historical data [76]
	Voltage	Load: normal [78,79], GMM [80]; Wind: Weibull [78,79], gamma [78]
PEM	Small signal	SGs: normal [81], binomial [55]; load: normal [55,81]; Wind: Weibull [82]
	Transient	Wind: normal [83], Load: normal [83,84]; SGs: normal [83] Fault CT: normal [84], Wind: Weibull [84]
	Voltage	Load: normal [85–92], historical data [93,94]; SGs: binomial [85] Power injection & line tripping: normal [95]; Electricity price: normal [89] Wind: Weibull [87–90,92,93], historical data [94]; Solar: beta [89,93], Weibull [88], historical data [90];
	Frequency	Load & EVs: normal [86], Wind: Weibull [86]
UT	Voltage	solar: normal [96], wind: Weibull [96,97]; Load: normal [96,97]; solar: beta [97]
LHS	Small signal	load: normal [98]
	Voltage	Wind: KDE [99], normal [100,101], Weibull [102] Solar: KDE [99], normal [100,101], beta [102]; load: normal [102]

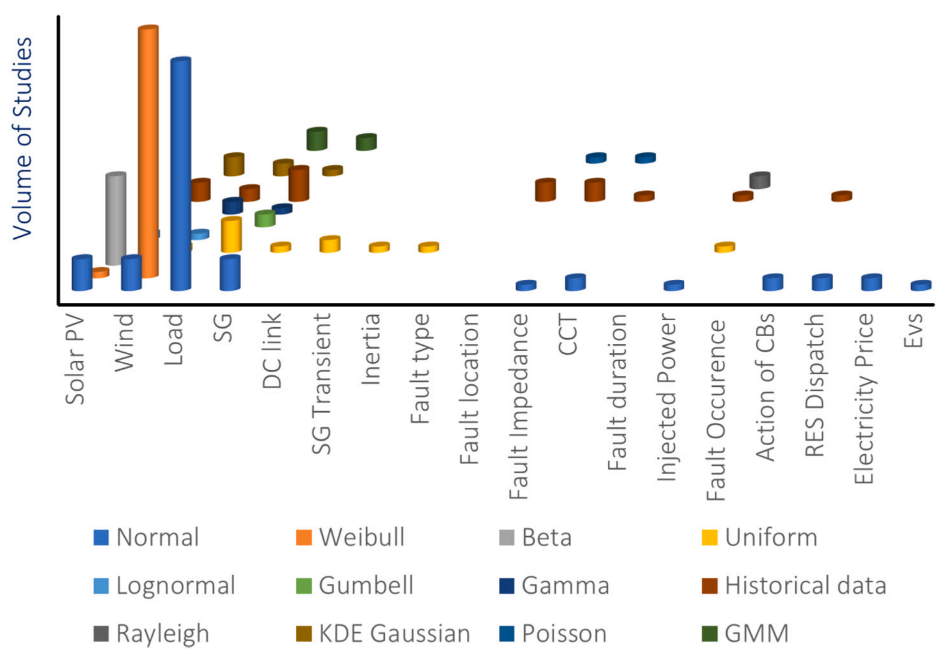

Fig. 2. A spectrum of system components whose uncertainties are probabilistically modeled.

3.2. Selection of journals for review

For a journal to be selected for review, first, it had to address the main concerns of the review and choice aimed at picking papers from high impact factor journals as rated according to JCR. Using keywords, different databases were queried and papers meeting the inclusion criteria were selected for review. The methodology used in this study is summarized in Fig. 3.

Google Scholar has been widely consulted as the central database of scientific literature. This was closely followed by IEEE Xplore, Web of Science, Scopus, Science Direct, IET, and National Renewable Energy Laboratories (NREL), among others. The inclusion and exclusion of a paper in the review followed a meticulous choice process, as shown in

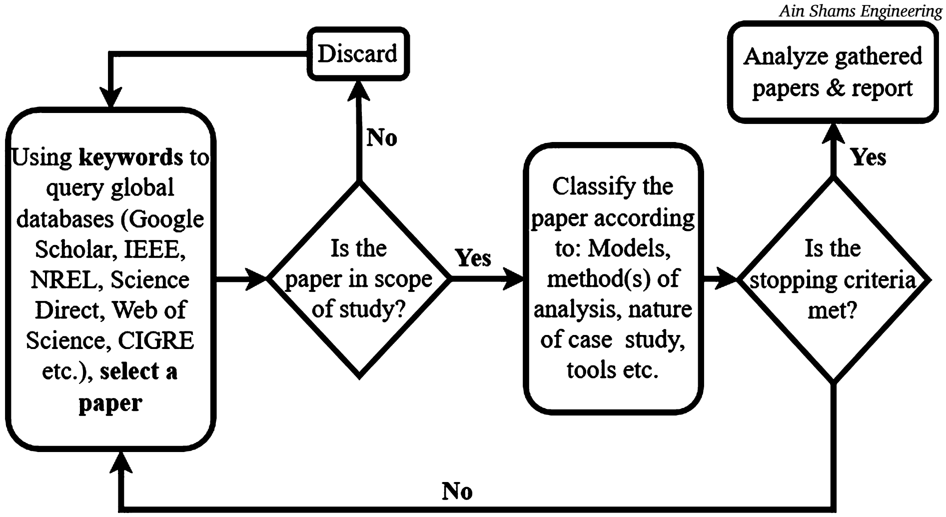

Fig. 3. Descriptive approach to the review.

Fig. 3. Although uncertainties regarding many physical systems have been studied, the exclusion/inclusion criteria aim to identify high quality literature that is particularly relevant and related to the analysis of modern power systems integrated with DERs.

4. Results and discussion of findings

4.1. Modeling of system RVs

Fig. 4 shows the prominent system RVs with their corresponding statistical models derived from the sampled literature provided in Table 2. As shown in Fig. 4, the modeling trend of the DERs and loads in power system studies tends to assign a single probability distribution function (PDF) to each system input RV. The RVs are predominantly modeled using specific distributions in the reviewed literature, with wind speed and load often assigned to Weibull and normal PDFs, respectively. Solar irradiance is commonly modeled using beta PDF. These trends are briefly discussed in the following subsections with the aim of assessing the suitability of these models.

4.1.1. Modeling of load demand with uncertainty

Conventional load modeling in static power system analysis typically employs a constant-power model, as shown by surveys in [34,35]. Consider the constant-power model for the load derived from the more general static exponential model defined in (1) and (2).

$$P_L = P_{LO}\left(\frac{V}{V_n}\right)^{\alpha} \tag{1}$$

$$Q_L = Q_{LO}\left(\frac{V}{V_n}\right)^{\beta} \tag{2}$$

where $\alpha, \beta = 0$ for a constant power load, P_{LO} and Q_{LO} are the active and reactive powers consumed at the rated voltage V_n; P_L and Q_L are the active and reactive powers consumed at the prevailing system voltage V.

While it is sufficient to represent the load using a constant power model in deterministic VSA, as defined in (1) and (2) with $\alpha, \beta = 0$, the prevalence of DERs in power systems has led to increased uncertainties necessitating PVSA, in which load uncertainties cannot be ignored. From Fig. 4, the statistical models used to study load uncertainty can be observed. The bar height represents the numerical prominence of the corresponding model. The surveyed literature predominantly models load using the normal PDF [62–67,85–92] among others, with a few using uniform PDF [42,43], or relying solely on historical data [76,94] for projecting future load expectations. Ref. [63] provides a mathematical uncertainty load model in each load bus i based on the normal PDF,

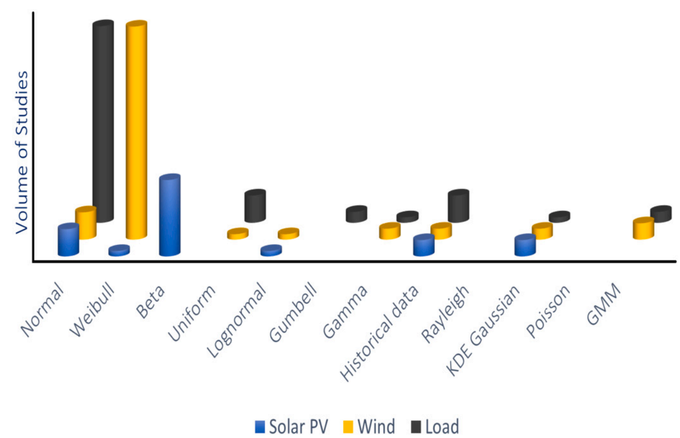

Fig. 4. Statistical models for the most prominent RVs.

f. Let ψ be the uncertain portion of the apparent load power with mean μ_ψ, and variance σ_ψ^2. The uncertain portion of the apparent load ψ, can then be modeled by a normal PDF defined in (3).

$$f(\psi) = \left(2\pi\sigma_\psi^2\right)^{-\frac{1}{2}} exp\left(-\frac{(\psi-\mu_\psi)^2}{2\sigma_\psi^2}\right) \tag{3}$$

Given the symmetrical nature of the load data at load buses, it is almost customary to model the load behavior using the normal PDF. However, as shown in Fig. 4, it is common to use the uniform PDF to represent the load. Suppose $\boldsymbol{\psi} = \{\psi_1, \psi_2, \dots, \psi_n\}$ is uniformly distributed; then, each member of $\boldsymbol{\psi}$ has equal probability. The uniform PDF f, over $\boldsymbol{\psi}$ is defined in (4).

$$f(\boldsymbol{\psi}) = \frac{1}{b-a} \tag{4}$$

where $a \le \boldsymbol{\psi} \le b$, and a and b are the lower and upper bounds of an uncertain RV $\boldsymbol{\psi}$. A uniform distribution is commonly applied when limited information is available regarding a specific uncertainty, owing to its simple form and ease of use.

Some studies, such as [71,80] have proposed the use of a Gaussian Mixed Model (GMM) for load uncertainty modeling. GMM assumes that an uncertain RV has spikes in a set of independent and identically distributed (i.i.d.) samples. Each sample has a slightly different mean and variance following a normal PDF. The overall distribution is the sum of individual normal PDFs [107]. However, GMM is increasingly being suggested for modeling and studying the correlation between uncertain RVs, including the system load.

The full load model combines the static load demand portion in (1) and (2) with the expected uncertain load portion to obtain (5) and (6):

$$P_L = P_{L0} + Re(E[\psi]) \tag{5}$$

$$Q_L = Q_{L0} + im(E[\psi]) \tag{6}$$

where $E[\psi]$ is the first moment of the uncertain portion of the load derived from any chosen model, such as in (3); $Re(E[\psi])$ is the real part; and $im(E[\psi])$ is the imaginary part of the apparent load demand ψ.

4.1.2. Modeling of wind speed and power

Wind turbine power generation (WTG) relies heavily on wind speed to the third degree, which is highly uncertain and varies stochastically on hourly, seasonal, and annual bases. Fig. 4 presents the prominent statistical models used in probabilistic power system studies to represent the wind speed. It can be deduced from Fig. 4 that wind speed v, uncertainty are prominently represented as a Weibull PDF [28,63,87–90,92,93] defined in (7). In the model, scale parameter c, represents the characteristic wind speed, which is proportional to the average wind speed. The shape parameter k, determines the dispersion of the wind speed data, with values ranging from 1 to 3. A larger k indicates a more constant wind speed and a high peaked PDF.

$$f(v) = \left(\frac{k}{c}\right)\left(\frac{v}{c}\right)^{k-1} exp\left\{\left(-\frac{v}{c}\right)^{k}\right\} \tag{7}$$

The Weibull PDF features prominently as a wind speed model because of its versatility in analyzing a wide range of data values, including non-Gaussian long-tailed datasets. However, some studies have explored alternative models, such as the normal PDF [100,101] and GMM [71]. However, it is crucial to note that wind speed data typically exhibit non-symmetrical, long-tailed, and non-Gaussian characteristics [108], making the use of normal and GMM PDFs prone to misfitting.

The power output of the WTG is then calculated as in [63] and is defined in (8).

$$P_w = \begin{cases} 0; & v < v_i \\ P_r * \left(\frac{v^3 - v_i^3}{v_r^3 - v_i^3}\right); & v_i < v < v_r \\ P_r; & v_r < v < v_0 \\ 0; & v > v_0 \end{cases} \tag{8}$$

where P_w is the WTG power output; P_r is its rated power; and v_i, v_r, v_0 are the cut-in, rated, and cut-out wind speeds, respectively.

4.1.3. Modeling of solar irradiance and power

The power output of the solar PV panels depends on the solar irradiance ξ. The models used in the literature to study solar irradiance are shown in Fig. 4. The most common approach for modeling uncertainties in solar irradiance is to consider ξ as a beta PDF, as applied in [65–68]. It is assumed that the beta distribution can fit a data-set of ξ received at any point, as defined in (9).

$$f(\xi) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\xi^{\alpha-1}(1-\beta)^{\beta-1} \tag{9}$$
$$\alpha, \beta > 0; \quad 0 < \xi < 1$$

where $\Gamma(.)$ is the gamma function, α and β are the shape parameters of the beta PDF. The beta PDF, although widely used in power system studies, as shown in Fig. 4, is limited to datasets with values between zero and one as defined in (9). However, solar irradiance values typically include zero (complete darkness), and can be greater than or equal to one. Using the beta PDF for studies often involves truncating the data such that it lies between zero and one, which can lead to analytical errors. Some researchers, such as [75], opted for a normal PDF. However, the normal PDF is not suitable for non-symmetrical irradiance data, and can potentially lead to analytical errors.

Another modeling approach gaining popularity for solar irradiance is the kernel density estimation (KDE) model [99]. The Kernel function (KF) employed in KDE must be symmetrical, with an area under the curve summing to one, and the density value must be non-negative [109]. A typical Gaussian KF for an RV ξ is defined in (10).

$$f(\xi) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{h\sqrt{2\pi}} * exp\left\{-\frac{(\xi-\mu)^2}{2h^2}\right\} \tag{10}$$

where μ represents the observed data point, ξ is the KF computation point, and h is the bandwidth, which is often estimated through optimization. Despite the growing popularity of KDE, it shares the limitations of the Gaussian PDF, which is most effective for symmetrical datasets. Applying KDE to non-symmetrical processes can lead to misleading results.

For skewed data with long tails, the lognormal distribution is preferred, and a few studies such as [63,64] have proposed its application for representing solar irradiance (ξ). This is favored because it can only fit positive data values starting from zero, and additionally, it can accommodate values greater than one, making it suitable for modeling irradiance. Solar irradiance, modeled with the lognormal PDF, has two key parameters: (a) μ, the location parameter that determines the PDF's position and represents the mean of the distribution, and (b) σ, the scale parameter that shapes the distribution and signifies its variance. Let ξ be the lognormally distributed solar irradiance with μ, and σ^2 [63]. The PDF of ξ is defined by (11) as follows:

$$f(\xi) = \frac{1}{\sqrt{2\pi\sigma^2\xi^2}} * exp\left(-\frac{(ln\xi-\mu)^2}{2\sigma^2}\right) \tag{11}$$

Estimating ξ through any chosen method allows the calculation of the solar PV output power as a function of the estimated ξ, [28]. Equation (12) defines the solar PV power output model, as follows:

$$P_{\text{PV}} = \begin{cases} P_r * \left(\frac{\xi^2}{\xi_{\text{std}} * \xi_c}\right); & 0 < \xi < \xi_c \\ P_r * \left(\frac{\xi}{\xi_{\text{std}}}\right); & \xi \geq \xi_c \end{cases} \tag{12}$$

where P_r is the rated power of the solar PV system, ξ_{std} is the solar irradiance under standard conditions (approximately $1kW/m^2$), and ξ_c is the specific irradiance at a point set to $0.15kW/m^2$ [110].

4.2. Probabilistic methods of analysis

Since the introduction of PM in 1974 by Borkowska in [111], various PM have been adopted to study various concerns of the power system. The list of system components has expanded, encompassing faults and new energy sources (Fig. 2). As such, different power system stability concerns have been studied considering these uncertainties. Fig. 5 presents an aggregation of power system stability studies, as extracted from Table 2. Of the reviewed literature, 51.52% focused on voltage stability, followed remotely by small-signal stability studies.

Consequently, a sample of the voltage stability studies was extracted and analyzed. Fig. 6 presents the distribution of different probabilistic methods applied in voltage stability studies. As expected, the MCS features prominently and is followed closely by the PEM. Therefore, in the following subsections, the solution structures of these prominent methods are provided with corresponding examination of their effectiveness and suitability for analyzing modern power systems.

4.2.1. Monte Carlo (MC) methods

This method is based on the law of large numbers, which states that an average tends to converge to its expected value if large samples are used in its determination. It is used to mirror the probability of a variety of outcomes, given the varied range of RVs inputs. MCS draws

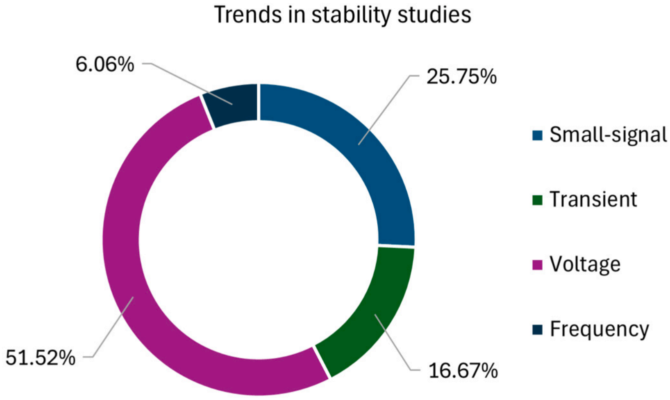

Fig. 5. Aggregation of power system stability studies.

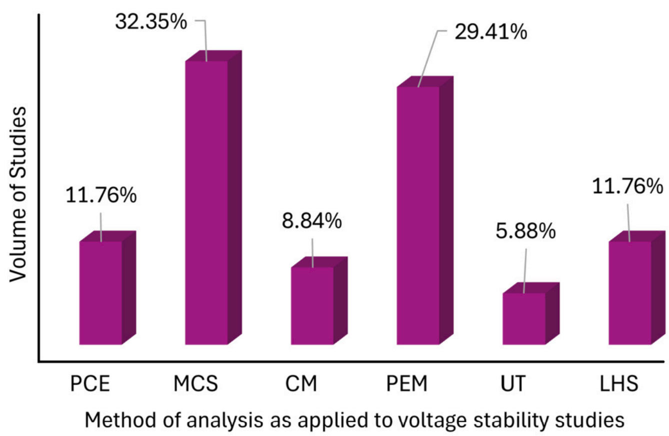

Fig. 6. Distribution of methods of analysis applied to voltage stability.

several random samples from an RV's assigned statistical distribution, runs the model with the drawn samples, and obtains the corresponding results. The procedure is repeated and the average parameter values are obtained as the expected values of an RV [110,112]. Randomness in value selection is used to mitigate bias. Extensions to improve MCS performance, which are widely applied in probabilistic stability analyses, include sequential MCS (SMC), quasi-Monte Carlo simulation (QMC), and non-sequential MCS (NSMC).

MCS method has been utilized in power system probabilistic stability studies since 1983 when it was used to study transient stability, as in [58–60,113]. It has since been applied to small-signal stability analysis, as in [55], and in [114], it was used to assess the voltage stability of the power system. MCS methods have been observed to exhibit a remarkable degree of accuracy and efficiency [115,116], however, they are computationally expensive and have a very slow rate of convergence, rendering their application only to the validation of other methods [117].

4.2.2. Latin hypercube sampling (LHS) methods

A square grid containing sample positions is a Latin square if and only if there is only one sample in each row and column. Latin hypercube generalizes this concept to multiple dimensions, ensuring that each sample is unique along every axis-aligned hyperplane to which it belongs [118]. This method involves stratifying the sample space into hypercubes. Random samples are drawn from these hypercubes and processed using a designed algorithm, leading to improved estimates of the uncertain parameters. LHS selects random samples progressively while keeping track of previous selections, ensuring faster convergence than MC methods [119].

The LHS method has been applied in probabilistic power system stability studies, for instance, in [98,117] to study the small-signal stability of a large power system with uncertainties, and in [102,120–122], it was used to assess the voltage stability of a large power network. In [117], a comparative study (between MCS and LHS) showed that the LHS method is more efficient, and faster than MCS. However, the LHS method suffers from dimensionality challenges for large systems, and its optimization criterion for uncorrelated data inputs has not been clearly articulated [123].

4.2.3. Polynomial chaos expansion (PCE) method

The PCE method aims to quantify system uncertainties by transforming deterministic system model equations into orthogonal polynomial functions of the RVs. Suppose $\boldsymbol{\zeta} = \left\{\zeta_1, \zeta_2, \ldots, \zeta_M\right\}$ is a set of i.i.d. samples of an RV $\boldsymbol{\zeta}$, such that its joint PDF, $f_\zeta(\boldsymbol{\zeta})$, defined in (13), is the product of the individual PDFs of ζ_i, that is,

$$f_\zeta(\boldsymbol{\zeta}) = \prod_{i=1}^{M} f_{\zeta_i}(\zeta_i) \tag{13}$$

For each single variable ζ_i and any two functions ϕ_1, ϕ_2 defined in the same sample space as $\boldsymbol{\zeta}$, a functional inner product maybe defined by the integral in (14) (provided it exists):

$$\langle \varphi_1, \varphi_2 \rangle = \int_M \varphi_1(\boldsymbol{\zeta})\varphi_2(\boldsymbol{\zeta}) f_{\zeta_i}(\boldsymbol{\zeta}) d(\boldsymbol{\zeta}) \tag{14}$$

Eqn. (14) defines the expectation $E\left[\varphi_1(\zeta_i), \varphi_2(\zeta_j)\right]$ with respect to $f_\zeta(\boldsymbol{\zeta})$. Two functions are said to be orthogonal with respect to the PDF, $f_\zeta(\boldsymbol{\zeta})$, if

$$E\left[\varphi_1(\zeta_i), \varphi_2(\zeta_j)\right] = 0$$

Using the above notation, a family of orthogonal polynomials $\left\{\phi_k^{(i)}, k\epsilon\mathbb{N}\right\}$ can be obtained as follows:

$$\left\langle \phi_j^{(i)}(\boldsymbol{\zeta}), \phi_k^{(i)}(\boldsymbol{\zeta}) \right\rangle = E\left[\phi_j^{(i)}(\zeta_i), \phi_k^{(i)}(\zeta_j)\right]$$
$$= \int_M \phi_j^{(i)}(\boldsymbol{\zeta})\phi_k^{(i)}(\boldsymbol{\zeta}) f_{\zeta_i}(\boldsymbol{\zeta}) d(\boldsymbol{\zeta}) = a_j^i \delta_{jk} \tag{15}$$

where δ_{jk} is the Kronecker defined as:

$$\delta_{jk} = \begin{cases} 0; & j \neq k \\ 1; & j = k \end{cases}$$

and a_j^i is the squared norm of $\phi_j^{(i)}$, i.e.:

$$a_j^i = \left\|\phi_j^{(i)}\right\|_i^2 = \left\langle \phi_j^{(i)}, \phi_i^{(i)} \right\rangle_i \tag{16}$$

Refs. [124] and [125] provided a match between different statistical models and their corresponding probable orthogonal functions. The PCE is typically applied using two approaches: Galerkin projections, where the model output is projected onto the space of polynomial basis functions, and collocation, where the PCE matches the model output for sampled input uncertainty values [124,126]. The collocation method is fully described in [127] as well as in other companion studies, such as [41], in which a demonstration of the method for studying system uncertainties is presented.

Whereas this approach is fairly recent, it is gaining popularity in engineering applications owing to its ability to handle a variety of stochastic variables and properties that can be easily described by PDFs [128]. However, for the method to work, two orthogonal functions must be defined, and their inner product should exist in the space of each RV. For symmetrically distributed RVs, the orthogonality requirement is easy to fulfill; however, for non-symmetrically distributed RVs, this becomes a challenging task. Moreover, although the method is efficient for small systems, it becomes computationally intensive as the number of RVs increases. Therefore, the PCE has limited scalability and application.

4.2.4. Point estimation methods

PEM have been developed over time with several variants. The dominant schemes among these are the *2m-* and *2m+1-concentrations*. The general solutions for these schemes are described in detail in [129]. The PEM, as developed by Hong, aims to use K points, also known as concentrations, to condense and represent an RV's statistical distribution data obtained from its representative first few central moments, namely, the mean, standard deviation, skewness, and kurtosis. By using these K points as inputs to a process affected by uncertainty in the input RV, the effect of uncertainty can be studied. Accordingly, for each RV x_i, process function F is evaluated K times. Therefore, if there are m RVs, function F will be evaluated Kxm or $Kxm+1$ times [85].

Compared to other members of the PEM family, such as the two-PEM that would require 2^K computations for each RV, Hong's variants have gained a reputation for good computation speed, especially for relatively small systems with few input RVs. However, as the number of system variables increases, the Kxm or $Kxm+1$ products become significant, requiring significant computational resources; therefore, they are limited in scalability to larger systems. Moreover, the method is based on central moments to study the system uncertainties. For RVs with central tendency, such as those that are normally distributed, the use of central moments is adequate to represent the system RV. However, for highly skewed RVs characterized by long-tailed distributions, the use of central moments tends to misrepresent the system uncertainties. Therefore, the 2m and 2m+1 PEM have gained popularity owing to their considerable speed when applied to relatively small systems at the expense of accuracy and scalability.

Method of moments

The method of moments (MoM) belongs to the general category of PEM. It estimates the desired parameters of a population by equating the sample data moments with their corresponding theoretically derived moments from the population's representative statistical model. Suppose $\zeta_1, \zeta_2, \ldots, \zeta_n$ are random data samples obtained from a population of an RV $\boldsymbol{\zeta}$ with PDF $f(\boldsymbol{\zeta})$. The k^{th} moment of the data sample is defined as in (17):

$$m_k = \frac{1}{n}\sum_{i=1}^{n} \zeta_i^k \tag{17}$$

The corresponding k^{th} theoretical moment of the population, which is a function of the population PDF parameters, can be calculated as in (18):

$$\mu_k = E[\boldsymbol{\zeta}^k] \tag{18}$$

Therefore, to obtain the estimators, the corresponding moments from the drawn samples in (17) and the population in (18) are equated, and the resultant equations are solved simultaneously. As such, the MoM maintains a simple approach to the estimation of uncertain parameters, can handle any statistical data distribution, and its solution can be generalized for any problem structure. Ref. [130] further demonstrated the application of the MoM in handling system uncertainties. It is evident from the developments in [130] that obtaining higher moments tends to be mathematically burdensome, and the method relies on approximations through, for instance, a Taylor series with truncation, resulting in estimator inefficiency. Additionally, the method heavily depends on the moments being defined, existing, and finite, which may not hold true for some distributions, especially heavily tailed ones [131].

Maximum likelihood estimation

Suppose that $\boldsymbol{\zeta} = \zeta_1, \ldots, \zeta_i, \ldots, \zeta_n$ is a vector of n random i.i.d. observations of an RV $\boldsymbol{\zeta}$. Suppose also that the probability of observing a single RV, ζ_i, $\pi(\boldsymbol{\zeta_i}|\theta_1, \ldots, \theta_k)$, depends on a set of k parameters, $\boldsymbol{\theta} = \theta_1, \ldots, \theta_k$. The likelihood of observing the n i.i.d. RVs is defined in (19) as

$$L(\boldsymbol{\zeta}|\boldsymbol{\theta}) = \prod_{i=1}^{n} \pi(\zeta_i|\theta_1, \ldots, \theta_k) \tag{19}$$

It was argued in [132] that if $\hat{\theta} = \hat{\theta}(\zeta)$ is a function that produces an estimate of $\boldsymbol{\theta}$, then the maximum likelihood estimation (MLE) of the parameter vector θ can be obtained as in (20).

$$\hat{\boldsymbol{\theta}} = \hat{\theta}(\boldsymbol{\zeta}) = argmax L(\boldsymbol{\zeta}|\theta_1, \ldots, \theta_k) \tag{20}$$

where $argmax(.)$ returns the vector $\boldsymbol{\theta}$ that maximizes the likelihood function $L(\theta_1, \ldots, \theta_k|\boldsymbol{\zeta})$. Therefore, the MLE obtains the most probable and optimal parameter points of the observed data, and is a good estimator of the desired parameter. The challenge of the MLE method is guaranteeing the attainment of a global maximum [131] for the estimator. Moreover, in seeking the optimal solution points, the MLE works through a burdensome evaluation of function derivatives, which increases the computational burden of the method when applied to large systems.

4.2.5. Bayesian parameter estimation

Suppose also that the desired parameters $\boldsymbol{\theta}$, defined under the MLE, of the observed data have a PDF, $\pi(\boldsymbol{\theta})$, also known as the prior. Then, with $L(\boldsymbol{\zeta}|\boldsymbol{\theta})$ as defined in (19), and from the definition of joint probability,

$$L(\boldsymbol{\zeta}|\boldsymbol{\theta})\pi(\boldsymbol{\theta}) = \pi(\boldsymbol{\zeta}, \boldsymbol{\theta}) = \pi(\boldsymbol{\theta}|\boldsymbol{\zeta})\pi(\boldsymbol{\zeta}) \tag{21}$$

Given the observed data ζ, the conditional distribution of $\boldsymbol{\theta}$ is:

$$\pi(\boldsymbol{\theta}|\boldsymbol{\zeta}) = \frac{L(\boldsymbol{\zeta}|\boldsymbol{\theta})\pi(\boldsymbol{\theta})}{\pi(\boldsymbol{\zeta})} \tag{22}$$

where

$$\pi(\boldsymbol{\zeta}) = \begin{cases} \int \pi(\zeta_i|\boldsymbol{\theta})\pi(\boldsymbol{\theta})d\boldsymbol{\theta} & \boldsymbol{\theta} \quad continuous \\ \sum \pi(\zeta_i|\boldsymbol{\theta})\pi(\boldsymbol{\theta}) & \boldsymbol{\theta} \quad discrete \end{cases} \tag{23}$$

Equation (22) is also known as Bayes' theorem; it combines the prior distribution with the likelihood function to obtain a posterior PDF of the RV over its parameters. Equation (23) defines the marginal probability of RV over the parameter spectrum [63]. The precise evaluation of (22) requires explicitly solving (23), which, in some cases, is complicated. However, since it is the parameter estimates that are needed and not the posterior PDF itself, Bayesian parameter estimation (BPE) has often proceeded by first seeking to evaluate the numerator of (22). This provides an analytical way of combining historical data with the statistical distribution of the RV to obtain more suitable parameter estimates. Then using a good sampling technique such as MCMC, Metropolis-Hasting, and importance sampling, among others, the parameters can be estimated using these numerical-based techniques. BPE enables obtaining better parameter estimates before they are submitted as inputs to a power system process. This ensures that better RV candidate estimates are used in the process and avoids repetitive regeneration of samples, as is the case with MCS and other PEM methods.

Therefore, the use of BPE appears to be more attractive owing to its low computational time, which allows for online applications and avoids dimensionality problems. Other notable attributes include: (a) incorporation of prior information to enhance the estimation accuracy, (b) generation of PDFs for parameter estimates, providing a comprehensive quantification of uncertainty, (c) utilization of diverse PDFs for modeling parameters, allowing flexible RV modeling, (d) capability to manage complex models challenging to estimate with alternative methods, and harness simulation techniques, such as MCMC to derive estimates from posterior distributions of RVs. However, only a few studies have utilized this approach [63].

4.2.6. Unscented transformation (UT) method

UT estimates an RV with a finite set of statistics using a nonlinear mathematical transformation via sigma points, which are weighted samples of the RV, to propagate the means and covariances of the nonlinear transforms. UT assumes that (a) it is easy to perform a nonlinear transform on a point and (b) it is possible to find a set of points in the state space that adequately represent the mean and covariance of the PDF of the RV. The basic idea of UT is presented in [133] and reintroduced as follows. Suppose $\boldsymbol{x} \in \mathbb{R}$ is an RV with PDF $f(x)$ and is subjected to a nonlinear transformation $\boldsymbol{g} : \mathbb{R}^n \to \mathbb{R}^m$ such that $\boldsymbol{y} = \boldsymbol{g}(\boldsymbol{x})$. The goal is to determine the moments of $\boldsymbol{y}$, particularly the mean $\boldsymbol{\mu}_Y$ and the covariances $\boldsymbol{\rho}_{YY}$, and $\boldsymbol{\rho}_{XY}$. The expectation of $\boldsymbol{y}$ is defined as follows:

$$\boldsymbol{\mu}_Y = \int \boldsymbol{g}(\boldsymbol{x}) f(\boldsymbol{x}) d\boldsymbol{x} \tag{24}$$

In practice, obtaining exact solutions for (24) is often challenging, necessitating the use of approximations. UT handles this by selecting a set of n sigma points $\chi_i, i = 1, 2, \dots, n$. Sigma points are weighted (ω_i) with the i^{th} sigma point corresponding to a weight. The sample mean and covariance matrix mirror those of $\boldsymbol{x}$. The transformation of sigma points using $\boldsymbol{g}$ yields $Y_i = \boldsymbol{g}(\chi_i)$. The expectation in (24) is approximated by the sample mean and covariance, using the following equations:

$$\hat{\boldsymbol{\mu}}_Y = \sum_{i=1}^{n} \omega_i Y_i \tag{25}$$

$$\hat{\rho}_{YY} = \sum_{i=1}^{n} \omega_i \left(Y_i - \hat{\boldsymbol{\mu}}_Y\right) \left(Y_i - \hat{\boldsymbol{\mu}}_Y\right)' \tag{26}$$

$$\hat{\rho}_{XY} = \sum_{i=1}^{n} \omega_i \left(\chi_i - \hat{\boldsymbol{\mu}}_X\right) \left(Y_i - \hat{\boldsymbol{\mu}}_Y\right)' \tag{27}$$

This method was first proposed by Uhlmann in [134], and has since been adopted as an upgrade to the extended Kalman and Kalman filter methods. In [135], its superiority over the extended Kalman filter method was demonstrated when two methods were deployed to study the dynamic-state estimation of the power system. UT also provides a method of studying nonlinear processes without the need for linearization [134].

The UT has been used in various system studies involving nonlinearities and uncertainties to assess the impact of system states. Ref. [136] used UT to predict the voltage collapse of a 10-bus system, in which the voltage collapse is framed as a function of the maximum loading of transmission lines, in [137–139] UT together with WAMS was proposed for improved power system state estimation because of its ability to handle a nonlinear system without the need for either linearization or calculation of the system Jacobian, whereas in [140], several modifications to the unscented Kalman transform were compared on a 140-bus to determine the most suitable for dynamic-state estimation. Ref. [141] applied UT to estimate the transient stability margin of power systems involving uncertainties arising from wind-power generation. The cited works demonstrated the superiority of UT over linear methods. However, the success of UT depends on selecting sigma points from the state space that can accurately propagate the means and covariances of nonlinear processes. However, deterministic sigma points may inadequately model the distribution of RVs [142].

4.2.7. Cumulants method

A detailed discussion of this method and its properties is provided in [143]. Consider that the r^{th} moment of a real-valued RV x is defined as:

$$\mu_r = E(x^r); \quad \forall r = 0, 1, \dots \tag{28}$$

If μ_r is finite and has a Taylor series expansion around the origin, the moment generating function can be defined as follows:

$$M(\varsigma) = E(e^{\varsigma x}) = E\left(1 + \varsigma x + \dots + \frac{\varsigma^r x^r}{r!} + \dots\right)$$

$$M(\varsigma) = \sum_{r=0}^{\infty} \frac{\mu_r \varsigma^r}{r!} \tag{29}$$

where the r^{th} moment is also the r^{th} derivative of $M(\varsigma)$ at its origin. The cumulants k_r are the coefficients in the Taylor series expansion of the cumulant-generating function about the origin defined as:

$$K(\varsigma) = log M(\varsigma) = \sum_{r} \frac{k_r \varsigma^r}{r!} \tag{30}$$

For which if $\mu_0 = 1$, then $k_0 = 0$. The first three cumulants can be determined as:

$$\begin{aligned} ck_1 &= \mu_1 \\ k_2 &= \mu_2 - \mu_1^2 \\ k_3 &= \mu_3 - 3\mu_2\mu_1 + 2\mu_1^3 \end{aligned} \tag{31}$$

The CM has been applied as an alternative to the MoM, in which determining moments, in some cases, might be difficult. Refs. [81] and [26] offer a summarized way of adopting the CM to handle system uncertainties and assess system sensitivity to changes in inputs. Ref. [144] developed a CM-based probabilistic method for simulating power systems in which Laguerre's polynomial functions were used to model the generation, load, and energy not served. The method has also been studied with other functions, such as Hermite polynomial, Gamma, and Gaussian distributions as in [145] for carrying out probabilistic optimal power flow and small signal stability analysis of power systems, among others. Whereas CM is simple to apply, its use is mostly restricted to PDFs with narrow standard errors in RVs and becomes computationally complex. Moreover, the use of the Taylor series expansion to linearize nonlinear processes with truncations results in significant errors in the estimates. Furthermore, whereas for small systems with fewer input RVs, CM appears to work excellently, it may have scalability limitations, and mathematical burdens with large systems having several RVs.

4.3. Key findings and emerging issues

4.3.1. Modeling of system RVs

The number of system components with uncertainties has increased since the introduction of PM to power system analysis, as shown in Fig. 2. However, over the years, the focus has steadily shifted toward DERs, particularly solar PV and WTG. Various statistical models have been developed to study uncertainties arising from these resources, with Weibull and normal distributions emerging as near-default models for wind speed and load uncertainty modeling, respectively. Table 3 summarizes the commonly used statistical models employed to study different uncertainties. The summary shows the common areas of application and what particular models would best be used to represent.

Solar irradiance, and consequently, solar PV power generation, appears to be the least studied among the three prominent system components. Researchers have employed a diverse range of models to study its natural occurrence and uncertain behavior, with the beta PDF gaining popularity. However, because of the limitation of fitting data between 0 and 1, which may not adequately represent solar irradiance data, alternative models, such as the normal and KDE PDFs, have been used. Nevertheless, these distributions are better suited for symmetrical datasets and may yield inaccurate results when applied to nonsymmetrical RVs.

4.3.2. Probabilistic methods of analysis

The MCS and LHS methods, collectively referred to as numerical-based PM, analyze system uncertainties by drawing large samples from the underlying PDFs of the RVs. They operate on the assumption that by drawing large data samples, the average of the observed data converges to the expected true values of the RVs, based on the principle of large numbers. This enables these methods to achieve high estimation accuracy. However, the need for a large sampling makes the solution algorithms of these methods generally time-consuming, requiring substantial storage space and high processing power.

Table 3
A concise summary of system RVs modeling.

Statistical Model	Definition	Usually applied to	Best suited for
Uniform PDF	$f(\xi)=\frac{1}{b-a}$ $a\leq\xi\leq b$	load, SG inertia, SGs, DC-link	RVs with limited data availability Simplified analytical studies needed
Beta PDF	$f(\xi)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\xi^{\alpha-1}(1-\beta)^{\beta-1}$ $\alpha,\beta>0;\quad 0<\xi<1$	Solar & wind speed	Not suitable for either solar or wind Studying probability of a CB tripping
Exponential PDF	$f(\xi)=\begin{cases}\lambda e^{\lambda\xi}; & \forall\xi>0\\ 0; & \forall\xi<0\end{cases}$	Load	Wind, load, solar, electricity price Its a general model that can be tuned to suit many RVs
Weibull PDF	$f(\xi)=\left(\frac{k}{c}\right)\left(\frac{\xi}{c}\right)^{k-1}exp\left\{\left(-\frac{\xi}{c}\right)^{k}\right\}$	Solar & wind speed	Wind speed Solar irradiance
Gaussian KDE	$f(\xi)=\frac{1}{\sqrt{2\pi\sigma^2\xi^2}}*exp\left(-\frac{(ln\xi-\mu)^2}{2\sigma^2}\right)$	Solar & wind speed	load data distribution Arbitrary data structures
Normal PDF	$f(\xi)=\left(2\pi\sigma_{\xi}^{2}\right)^{-\frac{1}{2}}exp\left(-\frac{(\xi-\mu_{\xi})^2}{2\sigma_{\xi}^{2}}\right)$	Load, electricity price, DERs dispatch, solar, CB tripping, wind, SGs, Power injection, EVs,	load data distribution, electricity price, fault CT, EVs, power injection,
Poisson PDF	$f(\xi)=\frac{e^{-\lambda}\lambda^{\xi}}{\xi!}$ $\xi=0,1,2,...;\quad\lambda>0$	Fault location	Fault location, equipment failure
Lognormal PDF	$f(\xi)=\frac{1}{\sqrt{2\pi\sigma^2\xi^2}}*exp\left(-\frac{(ln\xi-\mu)^2}{2\sigma^2}\right)$	Solar irradiance	Solar irradiance, maintenance data representation, failure rate, fatigue failure, etc.
Gamma PDF	$f(\xi)=\begin{cases}\frac{\lambda^{\alpha}\xi^{\alpha-1}e^{\lambda\xi}}{\Gamma(\alpha)}; & \forall\xi>0\\ 0; & \forall\xi<0\end{cases}$	Wind	Gust wind, load, solar, electricity price If $\alpha=1$ it becomes exponential, very versatile

The limitations of numerical-based PM have prompted the development of analytical-based approaches to address the computational burdens. Various analytical-based PM methods, such as PEM, PCE, UT, and cumulants, have been proposed as alternatives. For example, the PEM utilizes system data to obtain central moments, which are then concentrated onto representative point estimates. Therefore, the accuracy of PEM relies on unbiased estimators being obtained from error-free data. However, issues such as human data handling errors, measurement inaccuracies, and climate change effects can affect the credibility of the derived estimators. Moreover, the use of central moments leads to increased inaccuracies when the method is used to study systems with unsymmetrical data and experiences dimensionality problems with an increasing number of system RVs.

In addition, methods such as PCE and UT utilize representative surrogate functions by employing orthogonal functions and sigma points, respectively. The solution accuracy relies heavily on the careful selection of these surrogate functions, which is subjective to the solution designer. Moreover, for small systems, the computations are few and the method appears fast and efficient; however, as the system RVs increase in large systems, the computational requirements become increasingly prohibitive. Despite this, PCE, UT, and cumulant methods are noted for their faster solution algorithms, demanding fewer computational resources, such as storage and processing power.

A powerful analytical tool for modern power systems should blend the strengths of numerical and analytical approaches, utilizing large data samples for accurate estimates while mitigating computational burdens. BPE appears to fulfill this need by combining prior distributions and likelihood functions to derive posterior PDFs. Priors model historical data and knowledge of uncertain variables, whereas likelihood functions represent the theoretical system behavior. The parameters of the posteriors can be estimated using mature sampling techniques, such as MCMC algorithms, yielding comparatively accurate estimations of uncertain RV parameters.

From the surveyed literature, it was observed that many studies have been performed to consider different voltage stability concerns under various forms of system uncertainties. This was followed remotely by small-signal stability studies. Frequency stability was the least studied from the surveyed literature. Moreover, from the period considered (2012-2023), much focus has been placed on the participation of DERs in power systems. Very few studies have simultaneously considered the influence of DERs and disturbances on power system stability.

5. Conclusion

This paper presents a comprehensive state-of-the-art review on the modeling of power system components with uncertainties and the methods of analysis applied to DERs-integrated power systems. A thorough investigation of models for prominent system components with uncertainties is performed, with a specific emphasis on probabilistic methods (PM) from the perspective of power system stability analysis. PM is studied under two broad categories: numerical- and analytical-based PM. Acknowledging the limitations of numerical-based PM, analytical-based approaches have been explored as potential remedies. Recognizing the strengths and weaknesses of both numerical and analytical PM, the proposition that PM combining the aspects of both approaches may offer greater advantages is highlighted. Bayesian parameter estimation (BPE) is considered a promising choice in this regard; however, the method needs to be verified on different numerical examples. To date, only a few studies have used BPE. By delineating the salient features of models pertaining to system RVs and offering a comprehensive discussion of PM, this paper contributes pivotal insights into promising avenues for future research on power system modeling and analysis. The influence of DERs on power system stability appears to have been explored mainly from the view point of voltage stability. Further investigation of the influence of DERs and uncertainties on other forms of stability is required.

CRediT authorship contribution statement

Paul Wanjoli: Conceptualization, Formal analysis, Investigation, Methodology, Software, Validation, Writing – original draft, Writing – review & editing. **Nabil H. Abbasy:** Formal analysis, Investigation, Resources, Supervision, Validation, Visualization, Writing – review & editing. **Mohamed M. Zakaria Moustafa:** Formal analysis, Investigation, Resources, Supervision, Visualization, Writing – review & editing.

Declaration of competing interest

The authors have no conflict of interest whatsoever to declare.

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