A REVIEW OF PYTHAGORAS THEOREM AND ITS APPLICATIONS IN THE CONSTRUCTION INDUSTRY

BY

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Declaration

The work presented in this project is a result of my original work. Where I have used the works of other persons, due acknowledgements are clearly stated. It has never been submitted to any other university or institution of higher learning for the award of a qualification.

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Date:19/02/2021

Approval

The research work culminating in this project was conducted under my guidance and supervIs1on.

Name: Mr. RONALD KA TENDE

Signature: ……

Date:19/02/2021

Dedication

I dedicate the work in the project to my parents.

Acknowledgements

There are a number of people that I want to acknowledge for your support during this journey to complete this project. Each and every one of you is in my heart.

I begin by thanking God, without whom, none of this would have been possible.

I would like to thank supervisor Mr Ronald Katende for always giving me the time to discuss my ideas and progress and for providing your insight as to how to improve my work. Your prompt, professional advice and feedback was invaluable and truly inspiring, and certainly was a major reason as to why this project was completed successfully. I am truly blessed to have had the opportunity to work with you and receive guidance from such a helpful, intelligent man; I cannot thank you enough for everything.

I especially need to thank my family. To my parents: you have been there with me throughout everything, and have provided more love, support, encouragement and prayers than anyone could ever ask for or expect. The belief you displayed in me was the motivation and courage I needed and relied upon. Words cannot express my gratitude for everything you have given to me. To my sisters and brothers: thank you for sharing every moment of this journey with me and being my biggest fans.

To all my friends who supported me throughout this journey: know that your support has been vital and always sincerely appreciated. You have often been my rock, biggest supporters and advisors, whether you did so vocally or silently, and I truly am grateful. Whether I needed a push to go to the library or a person to vent to when I hit a road block, or help with editing, you were by my side and behind me with encouragement, time, smiles, solutions and most of all, love. I am truly blessed to have such amazing people around me. I love you all dearly, and while words cannot do justice to your efforts, rest assured that I appreciate each and every one of you for every single thing that you did for and with me.

Abstract

The famous theorem by Pythagoras defines the relationship between the three sides of a right triangle. Pythagoras Theorem says that in a right triangle, the sum of the squares of the two right-angle sides will always be the same as the square of the hypotenuse It states that "The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides". It is widely applied in solving problems in two, three dimensional figures and trigonometry and in construction industry.

The theorem is of fundamental importance in Euclidean Geometry where it serves as a basis for the definition of distance between two points.

In this study we give a brief historical overview of the famous Pythagoras' theorem and Pythagoras. We review a few selected proofs of the Theorem and discuss its applications in the construction industry. For example in surveying, carpentry, laying out squares and architectural designs.

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CHAPTER 1: INTRODUCTION

1.1 Historical Background of Pythagoras Theorem

Pythagoras theorem states that "The square of the hypotenuse of right triangle is equal to the sum of the squares of the other two sides". This theorem has remained one of the most famous geometrical theorems of all time, and has fascinated millions of people all over the world.

In mathematics, the Pythagoras theorem is a famous result in triangle geometry. This theorem is named after the Greek mathematician Pythagoras (570 BC- 495 BC), who by tradition is credited for its proof, although it is often argued that knowledge of the theorem predates him. There is evidence that Babylonian mathematicians understood the formula, although there is little surviving evidence that they used it in a mathematical framework. Pythagoras of Samos is often described as the first pure mathematician. He was an extremely important figure in the development of mathematics yet we know relatively little about his mathematical achievements. Unlike many later Greek mathematicians, where we have at least some of the books which they wrote, we have nothing of Pythagoras's writings. The society which he led, half religious and half scientific, followed a code of secrecy which certainly means that today Pythagoras is a mysterious figure. Whatever we know about Pythagoras is through the writings of other Greek contemporaries and later philosophers and mathematicians. Pythagoras is considered to be the founder of a group called the Pythagorean brotherhood. In fact it is believed that Pythagoras was

the first person to use the word mathematics.

1.2 Statement of the Problem

Pythagoras theorem is a fundamental theorem that is widely used especially when solving right angled triangles, solving problems involving three dimensions like cubes, cuboids. Despite its many applications, little attention has been paid to its historical background and proofs. The curriculum only emphasizes its statement and application.

1.3 Objectives

The following are the objectives of this study are:

(i) To study some of the selected proofs of the Pythagoras theorem based on dissection, sum of parts and proportions.

(ii) To analyse the practical applications of Pythagoras Theorem in the construction industry.

1.4 Scope of the Study

Out of five categories of proofs for Pythagoras theorem that is: Algebraic Proofs, Quarter ionic Proofs, Dynamic Proofs and other proofs are based on Calculus this study focuses only on Algebraic Proofs and Geometric Proofs. All the 10 proofs provided are based on dissection, sum of parts and proportion. The study also looks at the practical applications of the Theorem in the construction industry.

1.5 Significance of the Study

This project looks at the historical background and some selected proofs of the Pythagoras theorem and this will enable the learners to understand and apply it in real life situations. The project helps learners to appreciate the importance of Pythagoras Theorem its applications in the construction industry.

CHAPTER 2: LITERATURE REVIEW

2.1 Pythagoras

Pythagoras was Greek philosopher born about 572 B.C. on the Aegean Island of Samos off the coast of Asia Minor. In his early life he was a student of Thales. Thales had travelled in Egypt and learned much from the priests of Egypt, and he strongly advised his pupil, Pythagoras, to pay them a visit. Pythagoras heeded to this advice and travelled and gained a wide experience. This experience benefited him when he later had disciples of his own, and he became even more famous than his teacher. It is supposed that, besides travelling to Egypt, he travelled also to Babylon and perhaps on the Greek mainland. Returning home he found Samos under the tyranny of Polycrates and Ionia under the dominion of the Persians. He then migrated from Samos to Croton in Southern Italy in 530 B.C. There he lectured on philosophy and mathematics.

His lecture room was crowded with enthusiastic hearers of all ranks, and many of the upper classes attended. Women broke a law which forbade them to attend public meetings to hear him. Among them was Theano, the beautiful young daughter of Pythagoras' host, Milo. Pythagoras later married Theano, who wrote a biography of him. This manuscript was lost.

At the time of Pythagoras' arrival, Croton had suffered a crushing defeat by the hand of the Locrians. The moral and political reform which he promoted was evidenced by the fact that Croton was able to defeat and destroy the much more populous and powerful city of Sybaris just twenty years later in 510 B. C.

So remarkable was the influence of Pythagoras that the more attentive of his pupils gradually formed themselves into a society of brotherhood. This newly formed order, the Pythagorean Brotherhood, had much in common with the Orphic communities, which sought by rites and abstinences to purify the believer's soul and enable it to escape from the "wheel of birth." This new order was soon exercising a great influence across the Grecian world, though its influence was more religious than political.

Members of the society shared everything in common, held the same philosophical beliefs, engaged in the same pursuits, and bound themselves with an oath not to reveal the secrets and teachings of the school.

-,,., the course of time this order spread to other Italian cities. The order was most outstanding in me cities of Metapontum, Rhegium, and Locri. The order probably never ruled any of these cities directly, but rather exercised its influence through members who had attained leading political positions.

In time the influence and aristocratic tendencies of the brotherhood became so great that the democratic forces of southern Italy destroyed the buildings of the school and caused the society to disperse.

The first reaction against the Pythagoreans was led by Crlon. This action stemmed from the victory of Croton over Sybaris in 510 B. C. The civic disturbances which followed resulted in a setback to Pythagorean power in Croton. According to one report, Pythagoras fled to Metaponturn where he later died, maybe through murder, at the advanced age of seventy-five to eighty years.

An act of violence against the Pythagoreans worthy of mention was "the house of Milo" in Croton. Here fifty to sixty Pythagoreans were surprised and slain. Those who survived took refuge at Thebes and other places. The brotherhood, although scattered, continued to exist for at least two centuries longer.

2.2 Contributions of the Pythagoreans

Some of the important contributions of Pythagoras and the Pythagorean School were religion, theory of numbers, geometry, and astronomy.

One of the most advanced of the religious doctrines of the school was the theory of the immortality and transmigration of the soul. The moral and religious application which Pythagoras gave to the doctrine of transmigration continued to be the teaching of the school.

The pioneer work on discovery of the theory of numbers is attributed to the Pythagoreans. Aristotle said that the Pythagoreans "applied themselves to the study of mathematics and were the first to advance that science; in so much that, having been brought up in it, they thought that its principle must be the principles of all existing things. Pythagoras is said to have attached supreme importance to arithmetic, which he advanced and took beyond the realm of commercial use. He also made geometry a part of a liberal education, examining the principle of the science and treating the theorems from an immaterial and intellectual stand point.

Perhaps Pythagoras' greatest discovery was that of the dependence of the musical intervals on certain arithmetical ratios of lengths of string at the same tension, 2: 1 giving the octave, 3:2 the fifth, and 4:3 the fourth. This discovery must have contributed powerfully to the idea that "all things are numbers." According to Aristotle, the theory in its original form did not regard numbers as relations predictable of things, but as actually constituting their essence or substance. He said numbers seemed to the Pythagoreans to be the first things in the whole of nature, and they supposed the elements of numbers to be the elements of all things and the whole heaven to be a musical scale and a number. Later, in the fragmentary writings of Philolaus, things were spoken of, not as being numbers, but as having number and thereby becoming knowable. The development of these ideas into a comprehensive metaphysical system was probably the work of Philolaus. According to the Pythagoreans, the elements of numbers referred to by Aristotle were the odd and the even, which they identified with the limit and the unlimited. The unlimited and therefore the limit also, was conceived as spatial. Numbers were thus spatially regarded, and "one" was identified with the point, which was a unit having position and magnitude i "two" was similarly identified with the line; "three" with surface; and "four" with solid.

The odd and even and the limit and unlimited were, the first two of a set of ten fundamental oppositions postulated by the Pythagoreans. The remaining eight were the following: one and many, right and left, male and female, rest and motion, straight and curved, light and darkness, good and evil, and square and oblong. To the Pythagoreans the universe was in a sense the realization of these opposites.

Some further speculations of the Pythagoreans on the subject of number rested mainly on fanciful analogies. "Five" suggested marriage because it was the union of the first masculine and the first feminine number (3 + 2, unity not being considered a number); "one" was identified with reason because it was unchangeable; "two" with opinion because it was unlimited and indeterminate; "four" with justice because it was the first square number, the product of 11 equals Pythagoras pictured numbers as having characteristic. There were the triangular numbers, one, three, six, ten, and so on. Ten was known as the Holy Tetractys and was highly revered by the brotherhood. The triangular numbers were represented by figures of the following kind: which represent respectively one, three, six, and ten. The figures show at a glance the composition of the triangular numbers, for example 10 = 1 +2+ 3 + 4. To add a row of five dots to "ten" gave the next triangular number with 5 as the side, and so on, showing that the sum of any number of the series of natural numbers beginning with 1 was a triangular number. The sum of any number of the series of odd numbers beginning with 1 was similarly seen to be a square, so the square numbers were represented by figures like the.

Each of these square numbers could be derived from its predecessor by adding an L-shaped border. Great importance was attached to this border; it was called a gnomon (carpenter's rule). If the gnomon added to a square was itself a square number, e.g., 9, there resulted a square number which was the sum of two squares: thus 1 + 3 + 5 + 7 = 16 or 4^2, and the addition of 9 or 3^2 gave 25 or 5^2, thus $3^2 + 4^2 = 5^2$. Pythagoras himself was credited with a general formula for finding two square numbers the sum of which was also a square. Namely, (if m is any odd number),

$$m^2 + \left[\frac{1}{2}(m^2 - 1)\right]^2 = \left[\frac{1}{2}(m^2 + 1)\right]^2.$$

Letting m be a number of the form 2k + 1 where k is an integer, and then simplifying the formula, shows that it is an identity.

Other contributions to geometry by the Pythagoreans include the following : formulation of geometry, proving that that the sum of the three angles of any triangle was equal to two right angles, The discovery of the powerful method in geometry of the application of areas, and the discovery of the theory of proportion and the three means: arithmetic, geometric, and harmonic.

2.3 Early Developments of the Pythagoras Theorem

In as far as the development of the Pythagoras theorem is concerned the following people played a big role; the Greeks, Egyptians and Chinese.

According to Greeks, since Pythagoras' teaching was entirely oral and it was the custom of the brotherhood to refer all discoveries back to the revered founder, it is difficult to know just which mathematical findings and which philosophical viewpoints should be credited to Pythagoras, and which to the other members of the fraternity. However, tradition has unanimously ascribed to Pythagoras the independent discovery of the theorem which bears his name; namely, that the square on the hypotenuse of a right triangle is equal to the sum of the squares on the two legs. Others may have known of the theorem before Pythagoras, but he may well have given the first general proof of it.

The Egyptian geometrical knowledge seems to have been of a wholly practical nature. The Egyptians knew of special cases of the Pythagorean Theorem, but they didn't off er a general proof of it. An illustration of the way they used the theorem is given by the following example. The Egyptians were very particular about the exact orientation of their temples. They had to obtain a north and south line and an east and west line with accuracy. They observed the points on the horizon where a star rose and set, took a plane midway between them, and obtained a north and south line. To get an east and west line, which had to be drawn at right angles to this, they used a rope

The Chinese in the ·time of Chou-Kong had known of the Pythagorean Theorem. Although it was not enunciated in such a concise geometrical form as was given by Euclid, there can be no denying the fact that it was soundly established by the Chinese. The theorem was enunciated in the following words: Square the first side and the second side, and add them together; then the square root of the sum is the hypotenuse.

Again, when the square of the second side is subtracted from the square of the hypotenuse, the square root of the remainder is the first side.

Again, when the square of the first side is subtracted from the square of the hypotenuse 'lathe square root of the remainder is the second side.

2.

CHAPTER3: PROOFS OF THE PYTHAGOREAN THEOREM

The materials containing proofs of the Pythagorean Theorem are quite abundant, and in fact there exist some very large collections of the proofs of the theorem. For example, in the second edition of his book, The Pythagorean Proposition, E. S. Loomis has collected and classified 370 demonstrations of the famous theorem. These proofs are basically in four categories that is:

Algebraic Proofs - based on linear relations; Geometric Proofs based on comparison of areas; Quarter ionic Proofs based on vector operations and Dynamic Proofs based on mass and velocity. Other proofs are based on Calculus.

In this study, we shall look only the algebraic and geometric proofs which are further classified into dissection, sum of the parts, and proportions. In dissection proofs a figure of known area is dissected and the pieces reassembled in a different ways to give the desired areas, the proofs involving sum of the parts the area of the whole is equal to the sum of the areas of the parts and in proofs involving the Proportions the knowledge of similar triangles was employed.

Proof 1

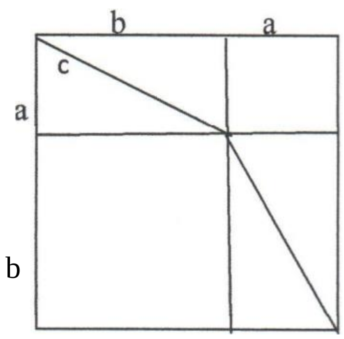


Figure 1

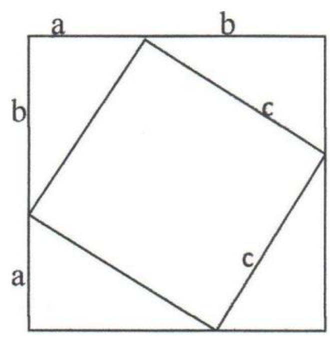


Figure 2

According to Eves [1], Let a, b, and c denote the legs and hypotenuse of the given right triangle, and consider the two squares in the accompanying figure, each having a + b as a side.

To prove that the central piece of the second dissection is actually a square of side c, employ the fact that the sum of the angles of a right triangle is equal to two right triangles. The rest of the proof is as follows:

The area of the square 1 is given *by,a? +b?*+4*()*

The area of square 2 is given by,c? + 4*('')*

Hense.a@ ++4(%)=e+4(%%)

Which gives: $a^2 + b^2 = c^2$

Proof 2

A second dissection proof was given by Bhaskara, the famous Hindu mathematician. In this proof the square on the hypotenuse was cut, as indicated in Figure 3, into four triangles each congruent to the given triangle and a square. The pieces were easily rearranged to give the sum of the squares on the two legs. The proof is as follows.

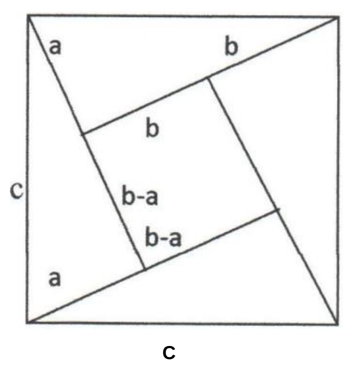


Figure 3

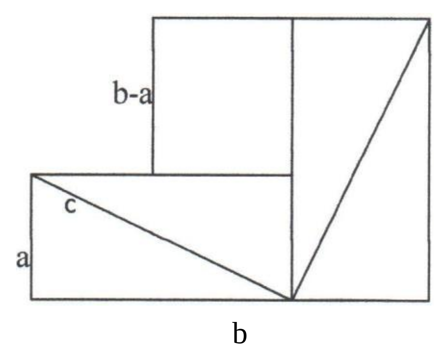


Figure 4

If c is the hypotenuse and a and b are the legs of the triangle, the area of the square in Figure 3 is c^2. The area of the figure formed by reassembling the pieces is,

4(")+-?=2ab +? 2ab +a° =a +

Therefore $a^2 + b = c^2$

Proof 3

very beautiful proof of the Pythagorean Theorem was given by General James A. Garfield. It appeared in the New England Journal of Education in 1876, five years before General Garfield tecame president. Garfield's proof utilizes the area of a trapezoid.

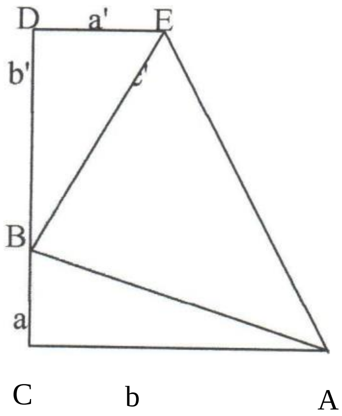


Figure 5

ABC is the given right-angled triangle. CB is extended to D, making b'=b. ED is constructed perpendicular to BD, making a' = a. BE and AE are drawn. The area S of the trapezoid CAED is given by the formula: $S = ¼(a + b')(b + a')$.

$s=7\overset{1}{a}+b)(b + a),$

Since $b' = b$ and $a' = a$.

$$S = \frac{1}{2}(a + 2ab + b?) = \frac{1}{2}a^2 + ab + \frac{1}{2}b^2,$$

Considering the areas at the triangles of the trapezoid,

$$S = \frac{1}{2}ab + \frac{1}{2}c^2 + \frac{1}{2}ab = ab + \frac{1}{2}c^2,$$

Therefore ab + kc? ±a?+ab+=y2 2 2 2

which gives $c^2 = a^2 + b^2$

3.

Proof 4

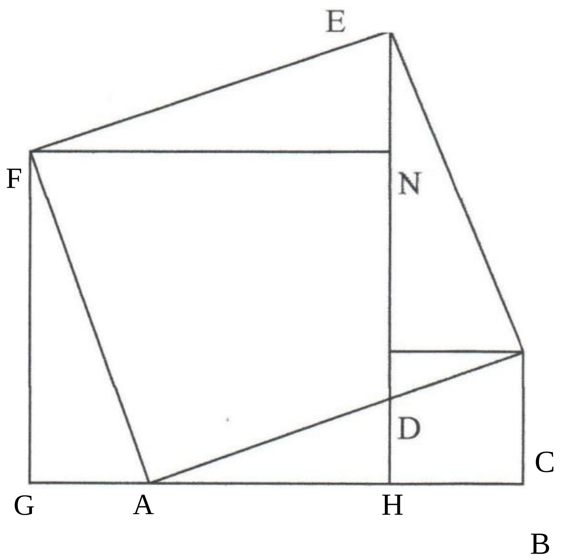


Figure 6

Figure ABC is a right-angled triangle. The four triangles ABC, AGF, FEN, and EDC are equal to each other. HNFG is a. square, and is equal to the square on AB.

(1) Area of GBCEF = *Ac*? + =*Ga* x*FG* +-*AB* x*BC*
2 2

2) Also area of *GBCEF* = GH+ =*EN* x *FM* +-*DC* x *ED* + *BC*?
2 2

Since triangles ABC, AGF, FEN, and EDC are equal:

FN 1- - 1- - 1- - 1- - *3)-GA* x *FG* +-*AB* x *BC*= -*EN* x
2 +-*DC* x *ED* 2 2 2

From (1) and (2) AC^2= GH^2+ BC^2Since *GH*= *AB*.

Then we have *AC*?= + *BC*?

Proof 5

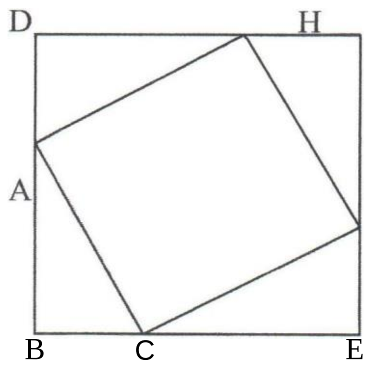


Figure 7

4.

I:: tight triangle ABC, BA is extended to P, making $\underline{AD} = \underline{BC}$; also BC is extended to E, making $\underline{CE} = \underline{AS}$, and the square is completed. A square is erected on AC. Then

$+ \underline{AD})' =$ area of square BEHD. But this area is composed of the area of the four triangles, hichare equal to each other, and the square of AC. Hence,

1» square BEID $= 4(\underline{29}) + \underline{Ac}°$

2Then $\underline{DB}? = \underline{(AB} + \underline{AD})? = \underline{AB} + 2\underline{(4} \times \underline{AD}) + \underline{AD}$?rom (l) and

(2),

3) $2\underline{(AB} \times \underline{AD}) + \underline{AC}^2 = \underline{AB}4\ 2\underline{(AB} \times \underline{AD}) + \underline{AD}^2$ Which

gives $\underline{AC}^2 = \underline{AB}^2 + \underline{AD}^2$

But $\underline{AD} = \underline{BC}$

Therefore: $\underline{AC}^2 = \underline{AB}^2 + \underline{BC}?$

Proof 6

Bbaskara also gave a second demonstration of the Pythagorean Theorem, which was rediscovered by John Wallis in the seventeenth century. In the following figure, the altitude his constructed on the hypotenuse c of the given right triangle.

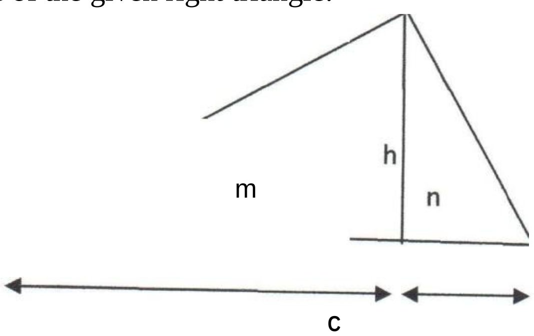


Figure 8

F= ·1 ah- 1 c b d c a rom s1mar nglt tr1angles,-
b m a n

This implies that $cm = b$ and $en = a^2$

Then by adding we have $a^2 + b^2 = c(m + n)$

7:erefore: $a^2 + b^2 = c^2$

Proof 7

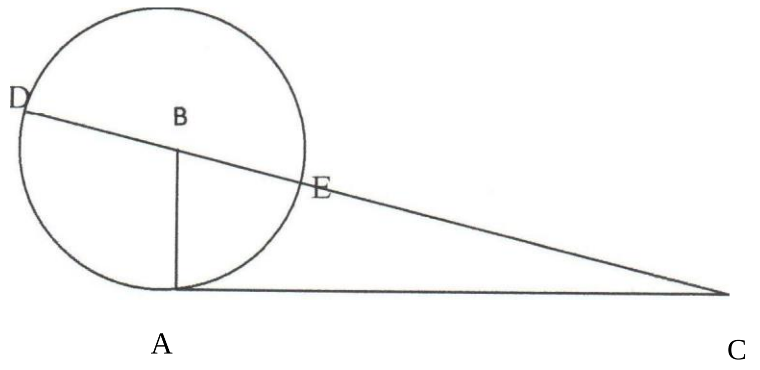


Figure 9

From an external point C, the tangent CA and the secant CD are drawn to the given circle having B as its center.

Therefore $\frac{EC}{AC} = \frac{AC}{DC}$

But $\underline{DC} = \underline{BC} + \underline{BD} = \underline{BC} + \underline{AB}$ Also

$\underline{EC} = \underline{BC} - \underline{BE} = \underline{BC} - \underline{AB}$

Therefore= $\frac{BC-AB}{AC} \quad \frac{AC}{BC+AB}$

which implies $\underline{BC}$?- = $\underline{AC}$? Finally

$\underline{BC}$? = $\underline{AB}$+ $\underline{AC}$?

Proof 8

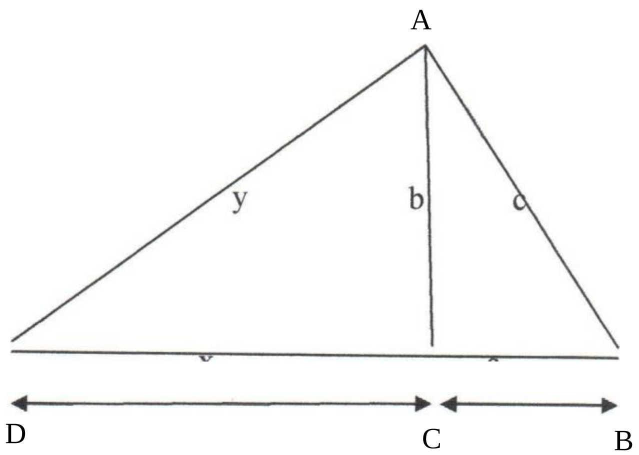


Figure 10

~ is a right-angled triangle. At A, a perpendicular is constructed to AB, and BC is extended ail it intersects this perpendicular at some point D. Now triangle DAB will also be a right angle, in which AC is an altitude. Let $\underline{AD} = y$, $\underline{DC} = x$, $\underline{AC} = b$, $\underline{BC} = A$ and $\underline{AB} = c$.

Therefore, by similar triangles

(1) $\frac{x}{b} = \frac{b}{a}$ or $b^2 = ax$

(2) Also $\frac{a+x}{c} = \frac{c}{a}$

(3) From (2) $c^2 = a^2 + ax$.

(4) From (1) and (3) $c? = a^2 + b?$

This can be stated as $\underline{AB}? = \underline{BC} + \underline{AC}?$

CHAPTER 4: APPLICATIONS OF PYTHAGORAS THEOREM

The Pythagoras' Theorem is a statement in geometry that shows the relationship between the engths of the sides of a right triangle. The right triangle equation is a + $b?$ = c'. Being able to fnd the length of a side, given the lengths of the two other sides makes the Pythagorean Theorem a useful technique used in the construction industry in the following ways:

Architecture and Construction

Given two straight lines meeting at right angles, the Pythagorean Theorem allows you to calculate the length of the diagonal connecting them. This application is frequently used in architecture, woodworking, and other physical construction projects. For instance, when building a sloped roof If the height of the roof is known and the length for it to cover, the Pythagorean Theorem can be used to find the diagonal length of the roofs slope. This information can be used to cut properly sized beams to support the roof, or calculate the area of the roof that would be needed to shingle.

The following are illustrative examples:

1. A mast is supported by two cables attached to the top, and to points on the ground 20m from its base. The height of the mast is 30m.

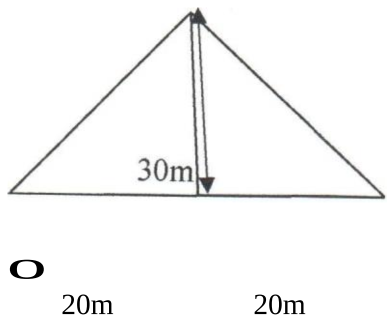


Figure 11

Calculate the length of the cables.

Solution:

Let the length of the cable be *l*

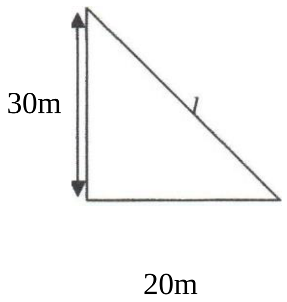


Figure 12

Applying the Pythagoras Theorem: $20^2 + 30^2 = l^2$ $400 +$

$900 = l^2$ $1300 =$

$l =$ V1300

$l =$ 36.056m

Therefore the length of each cable is 36.056m.

5. A roof of a house is to be constructed in form of a cone as shown on the picture, if the base has a radius of 1 Om and a = 26m, find the height *h* of the roof

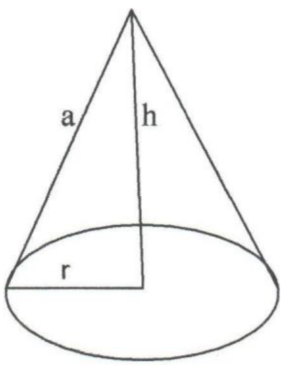


Figure 13

There is a right triangle formed by a, h, and r as the picture shows. The Pythagorean Theorem is stated for this triangle and used to solve for h.

r?+*h?*= *a*°

$10^2 + h^2 = a^2$

$100 + h^2 = 676$

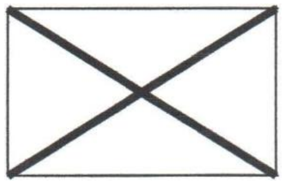
$h? = 576$

$h = \pm 24$

The negative value is ruled out, and so the height of the roof is 24m.

- A gate is strengthened by fixing a strut along each of its diagonals. The gate is 2 metres long and 1.2 metres high.

1.2 m



2m

Figure 14

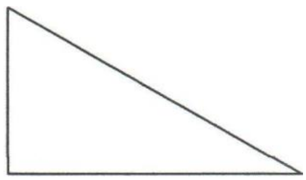
5md the length each strut.

Solution:

Ler the length of the strut be *l*.

~ s a right angle formed by the height and length of the gate and the each of the diagonals.

1.2 m



12

m Figure

15

Applying the Pythagoras Theorem: $2^2 + 1.2^2 = 4 +$

$1.44 = l^2$

$5.44 = [$

$l = \underline{V5.44}$

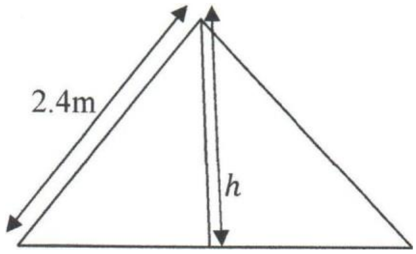
$l = 2.332\text{m}$

Therefore the length of each diagonal is 2.332m.

- A tent is 2.8 metres wide. Its sloping sides are 2.4 metres long. Calculate the height of the tent correct to 2 decimal places.

  Solution:

  Let the height of the tent be *h*.



2.8m

Figure 16

Applying the Pythagoras Theorem: $h^2 + 1.4^2 = 2.4^2$ $h^2 +$

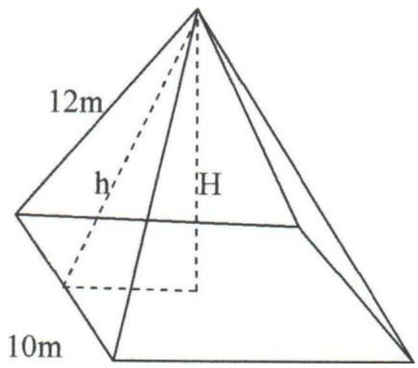
$1.96 = 5.76$ $h?=3.8$

1=V3.8

$l =$ 1.949m

Therefore the height of the tent is 1. 949m.

5. A roof of a house is to be made in form of a square-based pyramid as shown in the figure below. Its base has sides 1 Om long. The other faces of the pyramid are isosceles triangles with sides 10m; 12m; and 12m long.



10m

Figurel7

(:? Find the exact value ofh, the length of the height of the triangular face. b

Find the exact value ofH, the length of the height of the pyramid.

Solution: Let the vertices be labelled as shown in the figure below. The value of h can be found by stating the Pythagorean Theorem on the right triangle ABD.

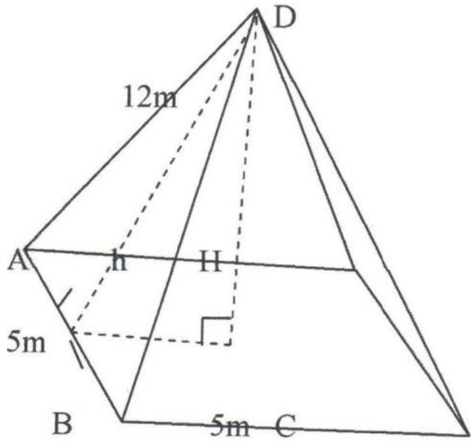


Figure 18

$$(Sm)^2 + h = (12m)^2 \ 25m^2$$

$$+ h^2 = 144m^2 \ h^2 = 119m^2$$

$$h = \pm 119m$$

Therefore, the height of the triangular face isv'l 19m. This 1s also calledtheslantheightofthepyramid.

b) Let use the labels shown on the figure above. We can find the value of H by stating the Pythagorean Theorem on the right triangle BCD. The hypotenuse is the side opposite the right angle.

(5m)? + H?= h?

25m? + H= (119m) 2

$$25m^2 + H? \ 119m^2 \ H^2 =$$

$$94m^2$$

H =V9an

The height of the Pyramid is /94m long

Laimg out Square Angles

- _ ~~orean Theorem is also used in construction to make sure buildings are square. A gs whose side lengths correspond with the Pythagorean Theorem - such as a 3 foot by 4 foot by 5 foot triangle- will always be a right triangle. When laying out a foundation, or ssrucung a square comer between two walls, construction workers will set out a triangle from -s:=gs that correspond with these lengths. If the string lengths were measured correctly, the -~site the triangle's hypotenuse will be a right angle, so the builders will know they are acting their walls or foundations on the right lines.

- ~ing out concrete footings for a new building, the Pythagorean Theorem is the most ct - +.. :=ethod available for making square 90 degree angles. It is the same as the old 3-4 - 5 a er trick, only more precise, because the exact comers can be located. One can apply the ties of the above Pythagorean triples.

Carpentry

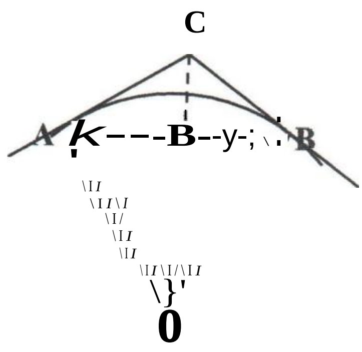
er area in construction where the Pythagoras Theorem is applied is carpentry. Modem .::c .. e=•~· work is so much easier when the Pythagorean Theorem is applied to the task at hand. framing, squaring walls, and foundations rely on this basic principle of mathematics.

Surveying

Surveying is the process by which cartographers calculate the numerical distances and heights eereen different points before creating a map. Because terrain is often uneven, surveyors must ~-s to take measurements of distance in a systematic way. The Pythagorean Theorem is used to calculate the steepness of slopes of hills or mountains. A surveyor looks through a esope toward a measuring stick a fixed distance away, so that the telescope's line of sight and measuring stick form a right angle. Since the surveyor knows both the height of the meas nng stick and the horizontal distance of the stick from the telescope, he can then use the mere to find the length of the slope that covers that distance and from that length, determine seep it is.

r gemetic design of highways, Pythagoras Theorem is used in determining horizontal and el *distances* of horizontal and vertical curves. Vertical curves are found at intersection of sices on a highway or a roadway and provide a safe and a comfortable ride for vehicles.

6.



a vertical curve

idO is radius of the vertical curve

mi D is the horizontal distance

-;; s the vertical distance

C are tangents to the vertical curve

.........- t.:.e above diagram both the horizontal and vertical distances can be obtained by use of the ~'T:25 Theorem.

eample considering triangle OAC, $OC^2 = OA^2 + AC^2$.

CHAPTER 5: DISCUSSION, ANALYSIS AND CONCLUSION

In this chapter each proof has been analysed following the fundamental property on which each was based. All the proofs have been categorised according to those which make use of dissection, sum of the parts, and proportions.

Dissection proofs

In dissection proofs a figure of known area is dissected and the pieces reassembled in different ways to give the desired areas. Proofs 1 and 2 are of this type:

Proof 1. Two congruent squares were dissected differently, and the Pythagorean relationship was obtained by setting the areas of the two squares equal to each other.

Proof 2. The square on the hypotenuse was dissected into four congruent triangles and a square. The pieces were reassembled to give the sum of the squares on the legs.

Proofs involving sum of parts

In the proofs involving Sum of the Parts, the area of the whole is equal to the sum of the areas of the parts. These proofs are numbered 3, 4 and 5. The Pythagorean relationship was obtained in proof number 4 by use of the parts of a trapezoid, while proof number 5 used the parts of a square. They all depended upon constructing the squares on the sides or upon constructing some of the squares on the sides together with some auxiliary lines to obtain the "parts." The differences of these proofs lie mainly in the manner in which the constructions were accomplished:

Proof 3. A trapezoid was constructed with one of the legs of the given right triangle as a base. The Pythagorean relationship was then derived by use of formulas for the area of a trapezoid and a triangle.

Proof 4. The square on the hypotenuse of the, given right triangle was constructed inward, and the square of one of the legs was constructed outward. The square of the remaining side was constructed so as to have vertices in common with the other two squares. The squares on the legs were divided into parts, the sum of whose areas is equal to the square on the hypotenuse.

Proof 5. Four congruent right triangles were constructed along the interior sides of a square with a square remaining in the middle. The Pythagorean relationship was obtained by setting the area of the original square equal to the sum of the areas of the four triangles and the square in the middle.

Proofs involving proportions

In proofs involving the Proportions, knowledge of similar triangles was employed. Proofs numbered 6 and 8 employed the same general diagram and the necessary proportions were derived by use of similar triangles. The two proofs differ only in the use of the similar triangles which were used to derive the necessary proportions. Proof 7 used the same diagram with the exception that it had another diameter constructed perpendicular to the first one.

Proof 6. An altitude was constructed to the hypotenuse of the given right triangle, and from similar right triangles proportions were derived which when simplified give the Pythagorean relationship.

Proof 7: A right triangle was formed by constructing a tangent and a secant to a circle from an external point together with a radius of the circle. Proportions which involved secants and tangents to a circle provided the Pythagorean relationship.

Proof 8: A right triangle was constructed upon one of the legs of a right triangle so that the two right angles of the triangles were adjacent. From similar right triangles, proportions were obtained which provided the Pythagorean relationship.

As already noted above, the Pythagoras' Theorem is a statement in geometry that shows the relationship between the lengths of the sides of a right triangle.

By applying the Pythagoras Theorem we can be able to find the length of a side, given the lengths of the two other sides. This makes the Pythagorean Theorem a useful technique used in the construction industry in the following areas: architecture and construction, surveying and carpentry.

The many applications of the Pythagoras Theorem in the construction industry make it an important area of interest for researchers to focus on.

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